

# REVIEW OF INDIRECT METHODS USED TO DETERMINE THE $^1S_0$ NEUTRON-NEUTRON SCATTERING LENGTH

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Calculations with realistic potential models show that a 1% change in the  $^1S_0$  nucleon-nucleon (NN) potential strength changes the scattering length by 30%. It is this high sensitivity that makes the  $^1S_0$  NN scattering lengths important in quantifying the amount to which charge symmetry is broken in the strong nuclear force. Of the three  $^1S_0$  NN scattering lengths, the value of the neutron-neutron scattering length  $a_{nn}$  is the most uncertain. A number of reactions that produce two neutrons with low relative energy have been used to determine  $a_{nn}$ . However, the two reactions that give  $a_{nn}$  with the least theoretical uncertainty are pion-deuteron capture ( $\pi^- + d \rightarrow n + n + \gamma$ ) and neutron-deuteron breakup ( $n + d \rightarrow n + n + p$ ). Curiously, the values obtained using these two reactions are significantly different. In this talk the experimental techniques and theory used to determine  $a_{nn}$  for each of these popular reactions will be reviewed. In addition, the results of the two most recent  $nd$  breakup experiments will be reported.

## 1. INTRODUCTION

Most observed charge-symmetry-breaking (CSB) effects can be explained as being due to the differences in the masses (QMD) and electric charges of the  $d$  and  $u$  quarks [ 1]. These differences are manifested by the mass splitting in hadronic isospin multiplets and in the values of the  $\rho$ - $\omega$  and  $\pi$ - $\eta$  mixing amplitudes within meson-exchange potentials [ 1].

The QMD leads to a difference in the masses of the neutron and proton and many other hadrons, and to a difference between the neutron-neutron ( $nn$ ) and proton-proton ( $pp$ )  $^1S_0$  scattering lengths,  $\Delta a = a_{nn} - a_{pp}$  [ 1]. Therefore, an experimental determination of the scattering length difference,  $\Delta a$ , gives a direct measure of CSB and can be related to the QMD. This high sensitivity of the scattering lengths to details of the nuclear force at the quark level also is reflected in nucleon-nucleon (NN) potential models that are based on meson-exchange phenomenology. For example, for most realistic NN potential models, a 1% change in the potential strength results in a 30% shift in the value of the calculated scattering length. This enormous sensitivity of the NN scattering length to the potential strength can be understood to first order using effective range phenomenology. For a square well potential, the fractional change in the scattering length due to a small change in the potential depth is given by

$$\frac{\delta a_{nn}}{a_{nn}} = 1.23 \left( \frac{a_{nn}}{r_{nn}} \right) \frac{\delta V}{V}. \quad (1)$$

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Using typical values of  $a_{nn} = 18.8 fm$  and  $r_{nn} = 2.8 fm$ , implies that a 1% change in the depth of the square well potential will result in about a 10% shift in the value of  $a_{nn}$ .

The  $a_{pp}$  has been measured directly to high precision (of order 0.01 fm) using two-nucleon scattering. However, there is a relatively large uncertainty of about  $\pm 0.4 fm$  in correcting the measured value of  $a_{pp}$  for electromagnetic ( $em$ ) effects, which are sizeable for this parameter. Consequently, the main uncertainty in determining the nuclear part of  $a_{pp}$  is due to the theoretical uncertainties in the factors used to relate the measured value of  $a_{pp}$ , which includes all  $em$  effects, to the purely nuclear part of the scattering length. The situation is quite different for  $a_{nn}$ . For technical reasons direct measurements of  $a_{nn}$  using free neutrons have never been successfully executed. Up to now, all determinations for  $a_{nn}$  have been based on studies of reactions with at least three particles in the exit channel [2, 3, 4, 5]. In this paper, the results from measurements of neutron-deuteron ( $nd$ ) breakup and pion-deuteron ( $\pi^-d$ ) capture are reviewed. Studies using these reactions were chosen for review because historically they have provided the most trusted determinations of  $a_{nn}$ . After the review of two recent kinematically complete  $nd$  breakup measurements, some concluding remarks are made and a list of recommended next steps presented.

## 2. SUMMARY OF $a_{nn}$ RESULTS AND METHODS

In this section the results for  $a_{nn}$  are summarized for studies done using  $nd$  breakup and  $\pi^-d$  capture measurements. This review will cover only experiments reported between 1964 and the present. In all cases  $a_{nn}$  was determined by fitting the measured cross section for the  $nn$  final-state interaction (FSI), which has a maximum value when the relative momentum of the two interacting neutrons is nearly zero. In the data analysis of these experiments, the relative energy between the two neutrons was typically between 0 and 500 keV. This wide integration range was required to obtain sufficient statistical accuracy in the  $a_{nn}$  determination. A graphical summary of the situation is shown in Fig. 1. The graph is divided into three sections. The left most section contains the results from cross-section measurements of kinematically incomplete  $nd$  breakup experiments. Five of the experiments reanalyzed by Tornow *et al.* [6] using modern theory are displayed as open circles. The middle section contains results from kinematically complete  $nd$  breakup experiments. The results from  $\pi^-d$  capture measurements are in the right section. The solid line represents the stastically weighted average of the data points for each reaction type from 1964 through 1997. The dashed line in the left section is the average of the reanalyzed data.

Some details of the experiments from which the data in Fig. 1 are taken are given in the tables in this section. Each experiment is categorized according to the number of kinematic quantities measured. The two broad types of experiments are kinematically complete (KC) and kinematically incomplete (KI). The experiments are tagged according to the number of measured kinematic parameters, because the level of the kinematic constraints imposed in the experiment can significantly impact the signal-to-background in the measurement and the theory used in the data analysis.

While the lists in the tables below are rather extensive, they are likely not all inclusive. The omission of an experiment of the type being reviewed here is not to be interpreted as the result of a data evaluation exercise but rather as an oversight on the part of the

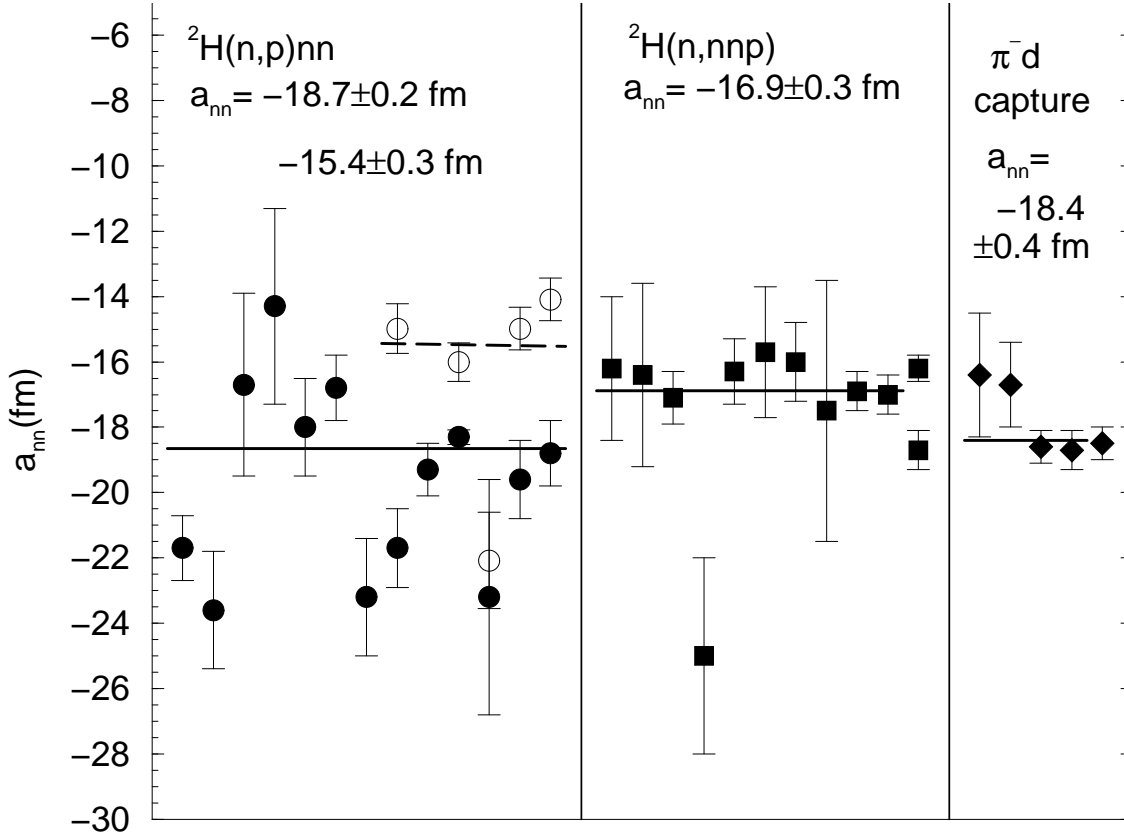


Figure 1. Summary of indirect determinations of  $a_{nn}$  from experiments reported between 1964 and the present. The data points for each reaction type are plotted on the horizontal axis in nearly chronological order and are as listed in Tables 1 and 3. Five of the experiments reanalyzed by Tornow *et al.* [ 6] are shown as open circles. The average values given at the tops of the sections for kinematically complete  $nd$  breakup measurements and the  $\pi^-d$  capture experiments don't include the results reported since 1997. The second average value in the left section is for the values obtained in the reanalysis [ 6].

author. The author apologizes for any such omissions. For each reaction type one or two experiments that reported relatively small error bars for  $a_{nn}$  are discussed for the purpose of presenting an overview of the experimental techniques employed and the theory applied in the analysis to determine  $a_{nn}$  from the measured cross sections.

### 2.1. $nd$ breakup

Details of the  $nd$  breakup experiments for which the results are plotted in the left and middle sections of Fig. 1 are given in Tables 1 and 2. Typically in the KI experiments the momentum vector of the emitted proton,  $\vec{p}_p$ , is measured. There are in the general sense two types of KC experiments. In the most common type the deuterium target is an active deuterated scintillator (DS). This type will be referred to as KC1. In these experiments the momentum vectors of both emitted neutrons and the energy of the emitted proton,  $p_{n1}$ ,

Table 1

Survey of  $nd$  breakup cross-section measurements that were used to determine  $a_{nn}$ . The survey covers the period between 1964 and the present. The main features of the analyses are given in Table 2.

ref.	year	$E_{lab}$ (MeV)	Measured Kinematics	Analysis Details	$a_{nn} \pm \Delta a_{ann}$ (fm)
[ 7]	1964	14.4	incomplete	3,9	$-21.7 \pm 1$
[ 8]	1965	13.9	incomplete	4	$-23.6 \pm 1.8$
[ 9]	1968	13.9	incomplete	2	$-16.7 \pm 2.8$
[ 10]	1967	14.0	incomplete	2	$-14.3 \pm 3$
[ 11]	1968	14.1	incomplete	2	$-18.0 \pm 1.5$
[ 12]	1968	8-28	incomplete	2	$-16.8 \pm 1.0$
[ 13]	1970	14.1	incomplete	4,9	$-23.2 \pm 1.8$
[ 14]	1972	50	incomplete	2,9	$-21.7 \pm 1.2$
[ 15]	1973	14.1	incomplete	2,9	$-19.3 \pm 0.8$
[ 16]	1973	14.1	incomplete	5,6,7,9	$-18.3 \pm 0.2$
[ 17]	1977	14.0	incomplete	6,7,9	$-23.2 \pm 3.6$
[ 18]	1986	49.6	incomplete	6,7,9	$-19.6 \pm 1.2$
[ 19]	1986	62.8	incomplete	6,7,9	$-18.8 \pm 1.0$
[ 20]	1969	14.1	complete	3,9,11	$-16.2 \pm 2.2$
[ 21]	1970	18.8	complete	1,9,11	$-16.4 \pm 2.8$
[ 22]	1973	18.4	complete	6,7,9,11	$-17.1 \pm 0.8$
[ 23]	1972	14.3	complete	2,9,11	$-25 \pm 3$
[ 24]	1974	18.4	complete	6,7,9,11	$-16.3 \pm 1.0$
[ 25]	1975	14.1	complete	6,7,9,11	$-15.7 \pm 2$
[ 26]	1974	14.2	complete	6,7,9,11	$-16.0 \pm 1.2$
[ 27]	1978	120	complete	1,9,11	$-17.5 \pm 4$
[ 28]	1979	17-21	complete	6,7,10,11	$-16.9 \pm 0.6$
[ 29]	1979	21-27	complete	6,7,10,11	$-17.0 \pm 0.6$
[ 4]	1999	13.0	complete	6,8,10,12	$-18.7 \pm 0.6$
[ 5]	2000	25.2	complete	6,8,10,12	$-16.3 \pm 0.4$

$\vec{p}_{n2}$ , and  $E_p$ , respectively, are measured. Sizeable experimental effects must be taken into account when comparing theory to data taken in KC1 experiments. The most important effects are: neutron attenuation in the DS, neutron multiple scattering in the DS, the energy dependence of the efficiency of the two neutron detectors and angle and energy averaging over the experimental acceptance. Some early high-precision KC1 experiments were done by B. Zeitnitz *et al.* [ 21, 22, 24]. The experimental setup is slightly different in the other type of KC experiment, which will be referred to as KC2. In KC2  $nd$  breakup experiments the deuterium target is a thin foil so that the momentum of the proton,  $\vec{p}_p$ , can be measured. In these experiments both  $\vec{p}_p$  and  $\vec{p}_{n1}$  are measured for each detected breakup event. As with KC1 experiments, the comparison of data taken in KC2 experiments to theory requires a number of effects to be taken into account: angle and energy averaging over the experimental acceptance, the energy and position dependence

Table 2

This table gives a short list of the main features of the theory used in the analysis of  $nd$  breakup data in studies designed to obtain a value for  $a_{nn}$  from the  $nn$  FSI cross-section enhancement.

Detail	Description
1	Watson-Migdal and effective range theory
2	Impulse Approximation, Pole Approximation
3	Born Approximation
4	Truncated graph method
5	Hybrid Final-State-Interaction theory
6	Faddeev theory
7	Separable nucleon-nucleon potential
8	Realistic nucleon-nucleon potential
9	$\ell = 0$ only NN partial waves
10	$\ell \geq 0$ NN partial waves
11	fit shape of $nn$ FSI enhancement only
12	fit shape and absolute cross section

of the efficiency of the neutron detector, and the detection correlation of the two emitted neutrons. An example of an early high-precision KC2 experiment was reported by von Witsch *et al.* [ 28, 29].

The results for  $a_{nn}$  from the two most recent KC experiments of González Trotter *et al.* [ 4], which was the KC1 type, and of Huhn *et al.* [ 5], which was the KC2 type, disagree by more than three standard deviations of the reported experimental uncertainties. This situation is a bit puzzling given that the data taken in both experiments were analyzed with the same theory.

As shown in Fig. 1 there has been a large number of kinematically incomplete  $nd$  breakup experiments over the last 35 years with the aim of determining  $a_{nn}$ . While each experiment has unique features, there are some common characteristics. The main common attributes of these experiments are: (1) the deuterium target is a deuterated polyethelene foil that is thin enough to allow the emitted protons to pass through with sufficient energy for detection, (2) a charged particle detector system is positioned to detect the protons from the breakup reaction that are emitted around  $0^\circ$ , and (3) the value of  $a_{nn}$  is obtained by fitting the enhancement in the proton-energy ( $E_p$ ) spectrum due to the  $nn$  FSI. The charged-particle detection system is usually either a magnetic spectrometer or a counter telescope with gas counters to eliminate the direct neutrons from the event trigger. The main technical challenge in these measurements is the reduction of protons from background sources, particularly in the flat part of the  $E_p$  spectrum. Even with more than ten kinematically-incomplete  $nd$  breakup experiments contributing to the effort, the result reported by Shirato *et al.* [ 16] dominates the computed average for this

type of measurement due to the small uncertainty in their value relative to that obtained in the experiments. A gas counter telescope was used in the experiment of Shirato *et al.* [16]. The main experimental issue in their measurement was the determination of the breakup protons from the lower energy part of their incident neutron beam. These contaminate protons affected the flat part of the  $E_p$  spectrum and had little influence on the  $nn$  FSI enhancement, which is at the extreme high end of the spectrum. In their analysis the  $E_p$  spectrum was fit with a two-term function by searching on three free parameters, the normalization constant for the term that described the flat part of the spectrum, the normalization constant for the term that described the  $nn$  FSI enhancement region of the spectrum, and  $a_{nn}$ .

The reanalysis of the  $E_p$  spectrum from some of the more recent kinematically incomplete experiments by Tornow *et al.* [6] gives very puzzling results. In all cases the magnitude of  $a_{nn}$  shifted to a substantially smaller value in the reanalysis with modern theory. On average the shift was about 3 fm out of 19 fm. The shift seems mostly associated with the discrepancy between the cross-section data in the flat part of the proton energy spectrum and the modern calculations. Because this part of the energy distribution is insensitive to the value of  $a_{nn}$ , the data were normalized to the calculations in this region. At incident neutron energies near 14 MeV the data were typically larger than the calculated cross sections. The consequence of this normalization is that the values of  $a_{nn}$  determined in the analysis of Tornow *et al.* [6] are on average lower than the original values.

## 2.2. $\pi^-d$ capture

The situation for determining  $a_{nn}$  from the  $\pi^-d$  capture reaction is summarized in Table 3. In these experiments a degraded pion beam is stopped in a liquid deuterium target. As in the case of the  $nd$  breakup experiments, the  $\pi^-d$  capture measurements also are divided according to the number of kinematic parameters measured.

Table 3

Survey of  $\pi^-d$  capture cross-section measurements that were used to determine  $a_{nn}$ . The survey covers the period between 1964 and the present.

ref.	year	Measured Kinematics	Analysis ref.	$a_{nn} \pm \Delta a_{nn}$ (fm)
[30]	1965	complete	[31]	$-16.4 \pm 1.9$
[32]	1975	complete	[31]	$-16.7 \pm 1.3$
[33]	1979	incomplete	[34]	$-18.5 \pm 0.4$
[35]	1987	complete	[34]	$-18.7 \pm 0.6$
[3]	1998	complete	[36]	$-18.5 \pm 0.5$

In KI measurements of the  $\pi^- + d \rightarrow 2n + \gamma$  reaction only the energy of the  $\gamma$ -ray ( $E_\gamma$ ) is measured. The measured  $E_\gamma$  spectrum is fit with theory to determine a value of  $a_{nn}$ . In the KC measurements, the momenta of the  $\gamma$ -rays and one of the neutrons

are measured for each detected capture event. The time-of-flight (TOF) spectrum of the detected neutron is fit to determine  $a_{nn}$ . The main experimental effects that must be taken into account when fitting the neutron TOF spectrum are the energy dependence of the neutron detection efficiency, the energy dependence of the attenuation and scattering of neutrons in the liquid deuterium target and the surrounding materials. The results from the two most recent KC experiments are in agreement within the reported uncertainties, which include a  $\pm 0.3$  fm due to theoretical uncertainties.

### 3. SUMMARY and RECOMMENDATIONS

The most popularly accepted value for  $a_{nn}$  comes from the  $\pi^-d$  capture measurements. The general belief is that these results are more reliable than those obtained from  $nd$  breakup or from other reactions in which there are three or more hadrons in the exit channel. This supposition is supported by the observation that the  $a_{nn}$  values obtained in the most recent high-precision  $\pi^-d$  experiments are in agreement while recent  $nd$  breakup experiments and analyses give discrepant results. Therefore, the recommended value for  $a_{nn}$  obtained using indirect methods is  $-18.6 \pm 0.3$  (experimental)  $\pm 0.3$  (theory) fm.

An important next step is to conduct  $nd$  breakup experiments for the purpose of resolving the discrepancies between the results from recent KC  $nd$  breakup experiments and for determining the cause of the shift in the value of  $a_{nn}$  in the reanalysis of the KI  $nd$  breakup cross-section data. While it is unclear whether investigating these problems will lead to a better determination of  $a_{nn}$ , this work will almost certainly strengthen our understanding of three-nucleon reaction dynamics in the kinematic region around the  $nn$  FSI.

The next sufficient step in this problem would be a direct measurement of  $a_{nn}$ . The high thermal neutron flux at the YAGUAR pulsed reactor in Russia opens opportunities for such measurements. The DIANNA collaboration is planning the first direct measurement of  $a_{nn}$  that use neutrons from a reactor. Some details of their proposed experiment are given in these proceedings.

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